Detection and Localization of Tones and Pulses using an Uncalibrated Array

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1 Introduction

This note describes a method for detecting weak narrowband signals (tones) incident on an array. This method is effective even if no information about the array geometry, element patterns, or instrumental gains and phases is available. In other words, this method achieves detection even if the array is uncalibrated. This property makes the method useful for *ad hoc* arrays consisting of large numbers of elements of different types and unknown responses, arranged in geometries which are arbitrary or unknown. Possible applications include SETI in large fields of view (FOV). A time-domain version of this same algorithm, suitable for detecting wideband pulses, is described toward the end of this note. This version of the algorithm may be suitable for detection of astronomical transients using, for example, LOFAR.

2 "Traditional" Method (BF)

A reasonable procedure for detecting a tone orginating from within the FOV of an N-element array is as follows:

1. Form N beams to cover the FOV.
2. FFT the output of each beam.
3. Compute the power spectral density (PSD) per beam (Simply, the squared-magnitude of each bin of the FFT).
4. Repeat steps 1–3 M times, averaging the PSDs. (Although M is perhaps traditionally 1 in SETI, we state the option of having $M > 1$ here to allow comparison with other methods later.)
5. Compute a detection metric for each FFT bin. The traditional detection metric is the PSD expressed in standard deviations σ from the mean across the remaining bins. Note that this requires *baseline removal*; i.e., compensation for the non-flat frequency response of the instrumentation.

6. A detection is declared if the detection metric for any bin of any beam is greater than some threshold. The direction of arrival is declared to be the pointing direction of the beam in which the detection metric is greatest.

We shall refer to this algorithm as "BF", short for "beamform-FFT". For large-N arrays, BF has some limitations:

- Step 1 requires that the array geometry be known and the receivers be well-calibrated to ensure that proper beams are placed on the sky.
- It is not possible to cover the entire sky with full beam gain when only N beams are available, because the crossover point between beams will be about -3 dB. Thus, the sensitivity for much of the sky is only half that acheived for the portions of the sky corresponding to beam maxima.
- Subtraction of the noise-only baseline from the computed PSDs is necessary to avoid biasing the detection metric, which degrades sensitivity. This an onerous task and difficult to do accurately in real-time.

3 Proposed Method – Version 1 (FXE)

The first version of the proposed method is referred to as "FXE", which is short for "FFT-Correlate-Eigenanalysis". The procedure is as follows:

1. Compute an L-point FFT individually for each antenna element.
2. For each FFT bin, compute the $(N^2 + N)/2$ non-redundant cross-products between elements.
3. Sum over M FFT outputs, where M can be as few as 2. This yields a "covariance matrix" $\mathbf{R}^{(l)}$ for each bin l of the FFT.
4. Compute $\lambda_1^{(l)}$, the primary (largest) eigenvalue of $\mathbf{R}^{(l)}$ for each bin. It is assumed that this will be done using Singular Value Decomposition (SVD), which yields the complete set of $min(M, N)$ eigenvalues and eigenvectors. However, SVD is computationally expensive, and an alternative will be considered later.

5. Compute $d_l = \lambda_1^{(l)} / (\mathrm{Tr}\{\mathbf{R}^{(l)}\} - \lambda_1^{(l)})$ for each bin. "$\mathrm{Tr}\{\mathbf{R}^{(l)}\}$" means "the sum of the eigenvalues of $\mathbf{R}^{(l)}$", or, equivalently, the sum of the diagonal elements of $\mathbf{R}^{(l)}$. In other other words, $\mathrm{Tr}\{\mathbf{R}^{(l)}\}$ is the total power intecepted by the array. The detection metric is d_l expressed in standard deviations σ from the mean across the remaining bins.

6. A detection is declared if d_l for any bin is greater than some threshold.

7. The steering vector associated with a detection is the eigenvector of $\mathbf{R}^{(l)}$ associated with $\lambda_1^{(l)}$ for the bin l in which the detection occurred. If the array is calibrated, then this steering vector also defines the direction from which the tone is incident on the array.

FXE overcomes the limitations of BF, as described below:

- It is not necessary to calibrate the array to achieve a detection. It is, however, necessary to calibrate the array to determine the direction from which the signal arrived. (In practice, it is anticipated that at least a rough calibration could be maintained, so that the signal could be localized in the sky using the primary eigenvector.)

- FXE is equally sensitive to the entire FOV, with no losses due to cross-over between beams.

- FXE is immune to baseline variations. This happens because the detection metric is is normalized to the noise power in a given frequenecy bin, and is independent of the noise power in other frequency bins. Thus, baseline subtraction is not required.

An additional bonus is that when SVD is used to compute a complete eigendecomposition, RFI can be mitigated in a simple and robust way. This is described in Section 7.

4 Proposed Method – Version 2 (FXE Lite)

An alternative version of the proposed method is "FXE Lite", so-named because it gives up some of the flexibility of the SVD to reduce the computational burden. In FXE Lite, the procedure is the same except that $\lambda_1^{(l)}$ and it's associated eigenvector $\mathbf{u}_1$ are computed using the Power Method [1]. The method is as follows:

1. Initialize $\mathbf{u}_1$ to be the sum of the snapshots used to form $\mathbf{R}^{(l)}$.
2. $\mathbf{q} \leftarrow \mathbf{R}^{(l)}\mathbf{u}_1$
3. $\mathbf{u}_1 \leftarrow \mathbf{q}/\|\mathbf{q}\|$
4. Repeat steps 2 and 3 P times.
5. $\lambda_1^{(l)} = \mathbf{u}_1^H \mathbf{R}^{(l)} \mathbf{u}_1$

This is an iterative method which yields very high accuracy, even with P on the order of M. The result is that primary "eigenpair" can be computed using on the order of N^2 operations, as opposed to N^3 operations required for the SVD. The disadvantages are that the method is not exact, and only the *primary* eigenpair is computed.

5 Performance Comparison

In this section, the performance of the various methods are compared using simulation. In this simulation, a single tone is incident on an array of $N = 16$ elements. The noise per element is independent and identically-distributed ("i.i.d.") white Gaussian noise. The FFT size is L =16384. Figure 1 shows the performance when the signal-to-noise ratio (SNR) is -13 dB (+29 dB within the bin). The performance is quantified in terms of σ obtained in the correct bin as a function of the number FFT blocks (M) processed. For the BF algorithm, the results from each successive PSD calculation are added to the sum of the previous results before computing the detection metrics. Thus, all methods are expected to improve with increasing M. Note also that no result is available for FXE with $M < 2$. In this example, we see that BF

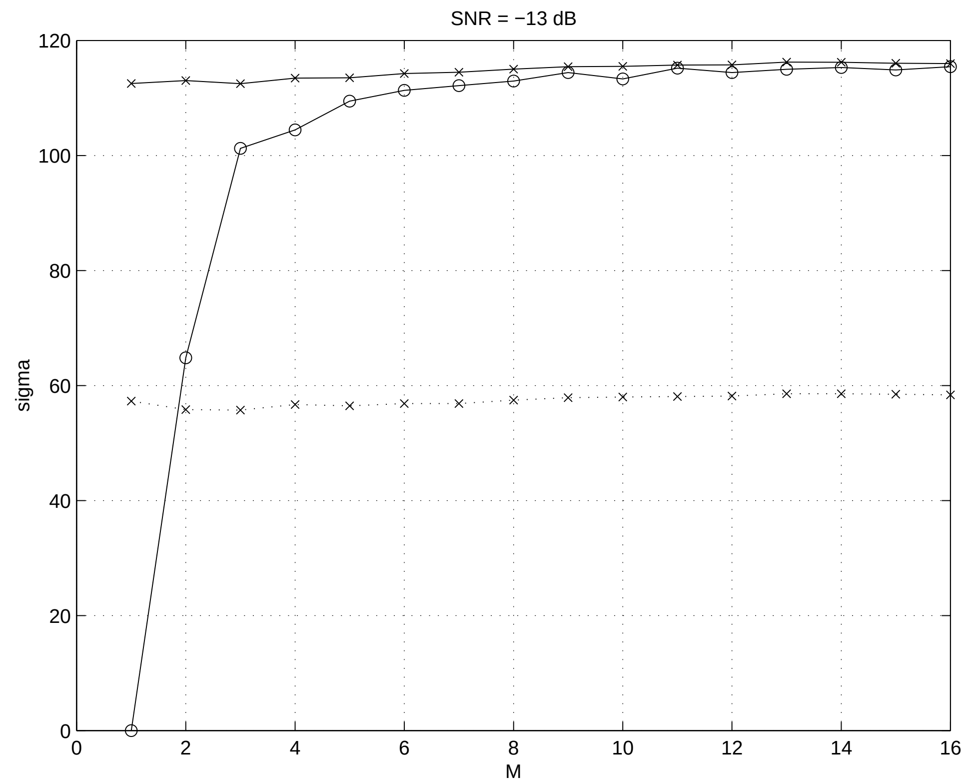

Figure 1: SNR $= -13$ dB. $\times$, *solid*: BF assuming full beam gain. $\times$, *dash*: BF assuming half-power beam gain. $\circ$: FXE.

significantly outperforms FXE for low M; however, within about $M = 5$ iterations, the performance of BF and FXE is about the same.

Also shown in Figure 1 is the detection performance for BF when the tone arrives from a cusp between beams. It is assumed that the cusp is at the half-power point, so the SNR delivered at the beam output is 3 dB less than the previous example in which we assumed that the tone's direction of arrival was coincident with the beam maximum. Note that in this case, FXE outperforms BF for every valid value of M.

Figure 2 shows the same experiment repeated for SNR$=-20.5$ dB. Here, we see that FXE starts off slightly worse than BF at half-maximum gain, but at $M = 8$ crosses over and by $M = N$ is approaching the performance of BF at maximum gain. This underscores an important point that FXE is in fact handicapped in it's detection performance by not having *a priori* pointing information. In other words, FXE's performance relative to BF suffers somewhat at low SNR because FXE is in some sense having to estimate a steering vector jointly with a detection metric,

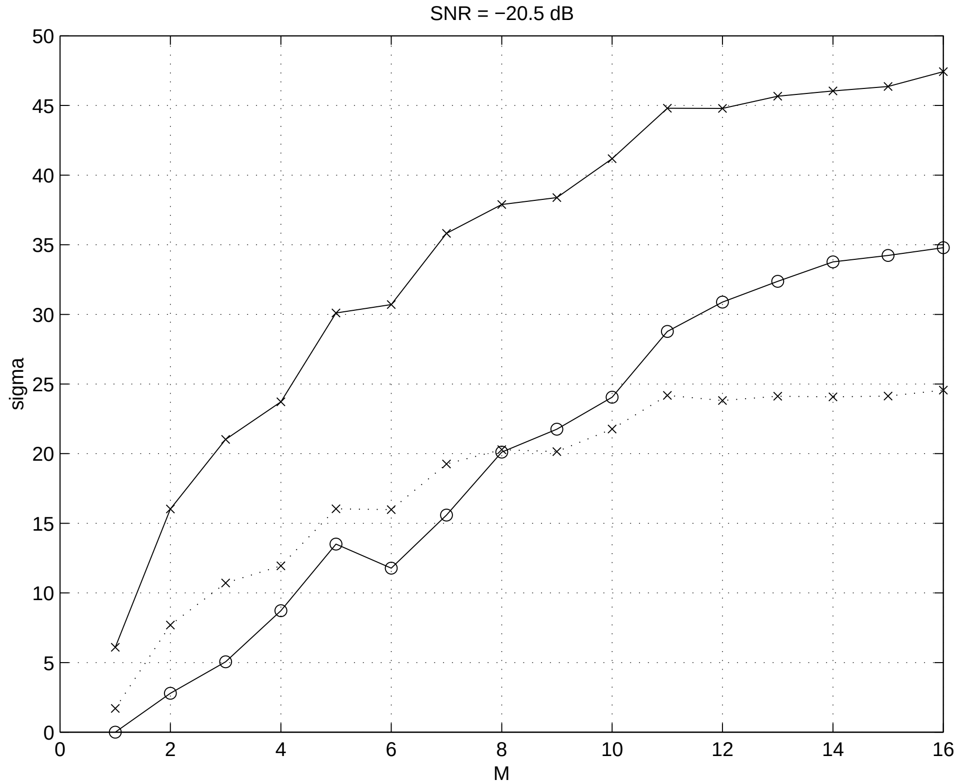

Figure 2: Same as Figure 1, with SNR equal to −20.5 dB.

whereas BF benefits from having the array gain provided with no effort required. Nevertheless, FXE performance still falls between that of BF at beam maximum and BF at beam crossover for sufficiently large M.

Figure 3 shows the same experiment, but now including FXE Lite with $P = M$. Note that FXE Lite does experience a slight degredation with respect to FXE, but still outperforms BF at the half-power points.

6 Computational Burden

In this section, we quantify the computational burden associated with the three methods described above. To this, we use the following rules for counting operation (FLOPs).

- A complex scalar plus a complex scalar is 2 FLOPs.
- A complex scalar times a complex scalar is 6 FLOPs.

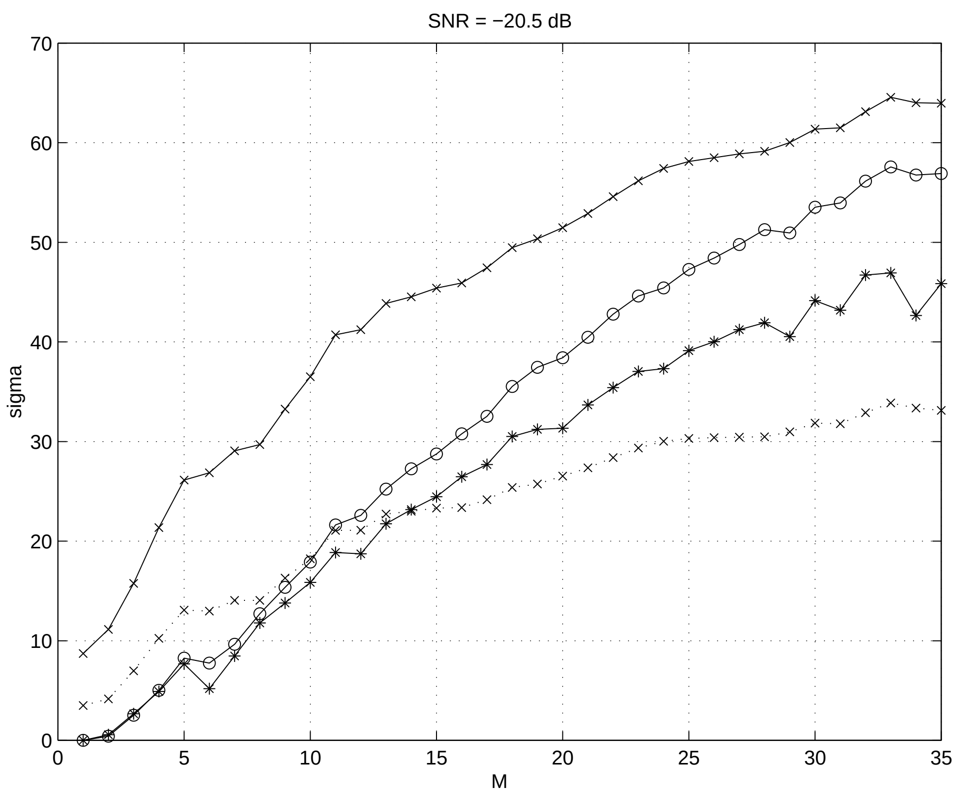

Figure 3: Same as Figure 2, now including FXE Lite (∗) and showing results for larger M.

- A complex FFT of length L is $6L \log_2 L$ FLOPs.
- An SVD of an $N \times N$ hermetian matrix is $40N^3$ FLOPs.

From these rules (in particular, the first two), we find:

- An inner product of 2 complex length-N vectors is $8N - 2$ FLOPs.
- The product of an $N \times N$ matrix with a vector is $8N^2 - 2N$ FLOPs.

Using these guidelines, we can compute the number of FLOPs required to process M blocks of LN samples from the array for a single iteration of detection. These estimates include all operations up to and including computation of the integrated PSDs (in BF) and computation of the primary eigenvalues/eigenvectors (in FXE and FXE light). The FLOP count for BF is:

$$8LMN^2 + 2(3M \log_2 L + M + 1)LN \tag{1}$$

and for FXE is:

$$40LN^3 + (4M - 1)LN^2 + (6M \log_2 L + 4M + 1)LN \tag{2}$$

and for FXE Lite is:

$$(8P + 4M + 7)LN^2 + (12P + 4M + 6M \log_2 L + 5)LN + 2(P + 1)L \tag{3}$$

The results for $L = 16384$ and $M = P = 5$ are shown in Figure 4. Whereas FXE requires orders of magnitude more FLOPs than BF, FXE Lite requires only slightly more FLOPs than BF. This is a reflection of the N^3 dependence of FXE compared to the N^2 dependence of BF and FXE Lite.

7 RFI Robust Operation

BF can be made robust to RFI by identifying the steering vectors associated with interferers and "projecting these out", i.e., modifying the beamformers so that the response for these steering vectors is zero. Details are given in [2] and [3].

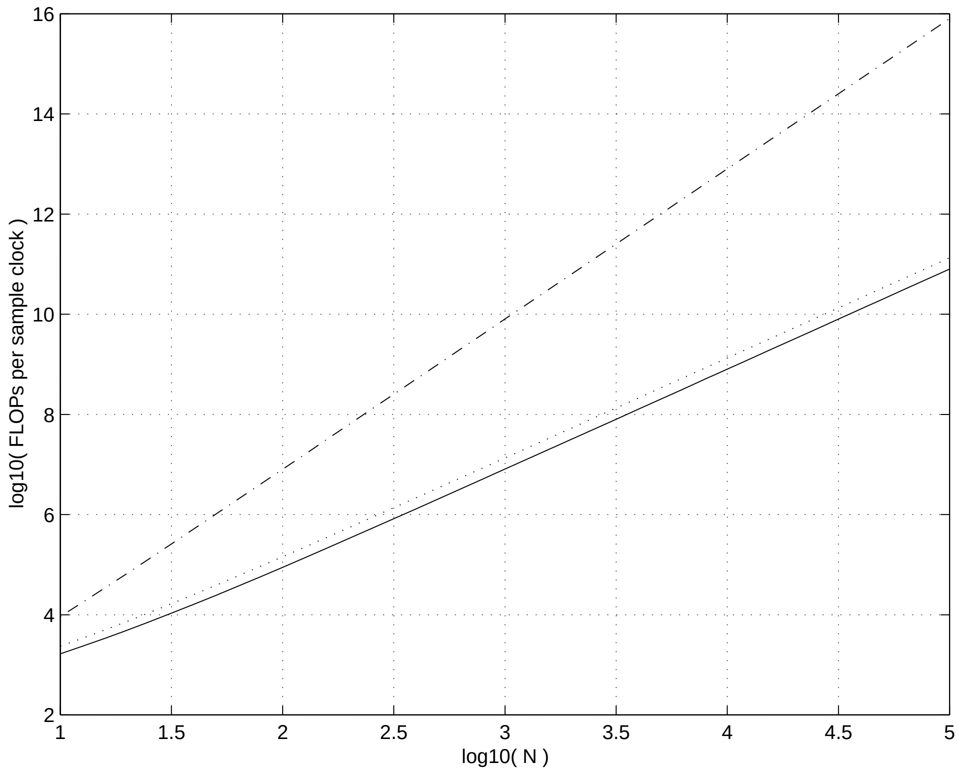

Figure 4: FLOPs per sample clock for each method as a function of number of antennas (N). *Solid:* BF, *Dash-Dot:* FXE, *Dot:* FXE Lite. $L = 16384$ and $M = P = 5$.

These same methods are applicable to FXE and FXE Lite. Instead of applying the projections to the beamforming coefficients, the projections are applied to the computed covariances. The computational cost is actually about the same as the analogous procedure for BF. A side benefit of this approach is that the associated pattern nulls are now broadband, because they are applied on a bin-by-bin basis.

For FXE (not FXE Lite), another possibility is as follows. If RFI is present in a bin, then the RFI then dominates the primary eigenpair. In this case, one simply uses $\lambda_2^{(l)}$ (the second largest eigenvalue) in lieu of $\lambda_1^{(l)}$, and the new detection metric becomes $d_l = \lambda_2^{(l)}/(\text{Tr}\{\mathbf{R}^{(l)}\} - \lambda_1^{(l)} - \lambda_2^{(l)})$. Note that this detection metric is completely independent of the RFI; in other words, we achieve detection simply by "ignoring" the RFI.

It may be possible to revise the power method (used in FXE Lite in lieu of the SVD) to allow FXE Lite to do similar tricks. This requires some more study.

8 Pulse Detection

Naturally-occuring transients are likely to be broadband. Here is a modification of the FXE strategy to handle this problem.

1. Divide the data record up into L blocks of M array output vectors (snapshots) each.

2. For each block l, compute the $(N^2+N)/2$ non-redundant cross-products between elements, summing over the M results per block. This results in L observations, each summarized by a covariance matrix $\mathbf{R}^{(l)}$.

3. Compute $\lambda_1^{(l)}$, the primary (largest) eigenvalue of $\mathbf{R}^{(l)}$ for each block, using SVD or the Power Method.

4. Compute $d_l = \lambda_1^{(l)}/(\text{Tr}\{\mathbf{R}^{(l)}\} - \lambda_1^{(l)})$ for each block. d_l expressed in standard deviations σ from the mean across the remaining blocks.

5. A detection is declared if d_l for any block is greater than some threshold.

6. The steering vector associated with a detection is the eigenvector of $\mathbf{R}^{(l)}$ associated with $\lambda_1^{(l)}$ for the block l in which the detection occurred.

References

[1] G.H. Golub and C.F. Van Loan, *Matrix Computations*, 3rd. Ed., The Johns Hopkins University Press, 1996.

[2] S.W. Ellingson and G.A. Hampson, "A Subspace-Tracking Approach to Interference Nulling for Phased Array-Based Radio Telescopes", *IEEE Trans. Antennas and Propagation*, accepted, scheduled to appear in the January 2002 issue. Download available: http://esl.eng.ohio-state.edu/rfse/downloads/aps9911.pdf.

[3] S.W. Ellingson and W. Cazemier, "Efficient Multibeam Synthesis with Interference Nulling for Large Radio Telescope Arrays", *IEEE Trans. Antennas and Propagation*, Submitted June 2000, revised July 2001. Download available: http://esl.eng.ohio-state.edu/rfse/downloads/aps0010.pdf.