# Detection of Tones and Pulses using a Large, Uncalibrated Array

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## I. Introduction

A problem of growing interest in the radio astronomy community is that of how to detect transient astronomical signals over a wide field of view (FOV). Two classes of signals of particular interest are ultra-narrowband tones, which are sought in SETI [1]; and wide-band pulses associated with (for example) gamma ray bursts [2] and intermittent "giant pulses" from pulsars [3]. A useful method to detect weak signals over a wide FOV is to use an array consisting of elements whose patterns are individually broad enough to cover the entire FOV, yet which can be processed together to achieve the sensitivity of a large, fixed-aperture antenna. The optimal detection algorithm for such a system requires a search over all possible directions of arrival (DOAs) [4]. However, this strategy is computationally expensive and is awkward to implement in practice due to the need to calibrate the array. A suboptimal strategy that removes the calibration requirement is simply to examine the eigenvalues of the spatial covariance matrix, using, for example, the Minimum Description Length (MDL) algorithm [5]. However, MDL (and related algorithms) typically perform poorly when receiver noise power, which may vary significantly between receivers, dominates over environmental noise – as is the case in astronomical observations using large numbers of inexpensive receivers.

This paper presents a simpler, suboptimal approach that avoids the need to calibrate the array, but is robust to noise covariances which are spatially or spectrally non-white. The tone-detection version of the algorithm is described in Section III, followed by a study of performance and computation burden in Sections IV and V, respectively. A pulse-detection version of this algorithm is outlined in Section VI. We begin by presenting a simplified version of the optimal spatial search strategy, to facilitate comparison with the proposed algorithm.

## II. Spatial Search Strategy (BF)

To detect a tone originating from within the FOV of an $N$-element array, the steps are:

1. Form a basis set of $N$ beams to cover the FOV.
2. FFT the output of each beam.
3. Compute the power spectral density (PSD) of each beam (Simply, the squared-magnitude of each bin of the FFT).
4. Repeat steps 1–3 $M$ times ($M \geq 1$), averaging the PSDs.
5. Compute a detection metric for each FFT bin. The traditional detection metric is the PSD expressed in standard deviations $\sigma$ from the mean across the remaining bins. Note that this requires *baseline removal*; i.e., compensation for the non-flat frequency response of the system.
6. A detection is declared if the detection metric for any bin of any beam is greater than some threshold. The DOA is declared to be the pointing direction of the beam in which the detection metric is greatest.

We shall refer to this algorithm as "BF", short for "Beamform-FFT". BF is optimal except for the use of a basis set of beams (as opposed to an exhaustive search over the continuum of all possible DOAs) and the use of the FFT (as opposed to an exhaustive search over the continuum of all possible frequencies). However, BF requires that the array geometry be known, and that the receivers be calibrated to ensure well-formed beams. Furthermore, BF does not cover the entire sky with full beam gain, since only $N$ discrete beams are available; in fact, the crossover point between beams will typically be at least $-3$ dB below maximum beam gain.

## III. Proposed Tone Detection Algorithm (FXE)

The proposed method is referred to as "FXE", which is short for "FFT, Correlate, Eigenanalysis". The procedure is as follows:
1. Compute an $L$-point FFT individually for each antenna element.
2. For each FFT bin, compute the $(N^2+N)/2$ non-redundant correlations between elements.
3. Sum over $M$ FFT outputs. This yields a covariance matrix $\mathbf{R}^{(l)}$ for each bin $l$ of the FFT.
4. Compute $\lambda_1^{(l)}$, the primary (largest) eigenvalue of $\mathbf{R}^{(l)}$ for each bin. It is assumed that this will be done using Singular Value Decomposition (SVD), which yields the complete set of $min(M, N)$ non-zero eigenvalues and associated eigenvectors. However, SVD is computationally expensive, and an alternative will be considered later.
5. Compute $d_l = \lambda_1^{(l)}/(\mathrm{Tr}\{\mathbf{R}^{(l)}\} - \lambda_1^{(l)})$ for each bin. "$\mathrm{Tr}\{\mathbf{R}^{(l)}\}$" is the sum of the diagonal elements of $\mathbf{R}^{(l)}$, which is the total power intercepted by the array. The detection metric is $d_l$ expressed in standard deviations $\sigma$ from the mean across the remaining bins.
6. A detection is declared if $d_l$ for any bin is greater than some threshold.
7. The steering vector associated with a detection is the eigenvector of $\mathbf{R}^{(l)}$ associated with $\lambda_1^{(l)}$ for the bin $l$ in which the detection occurred. If the array is calibrated, then this steering vector also defines the direction from which the tone is incident on the array.

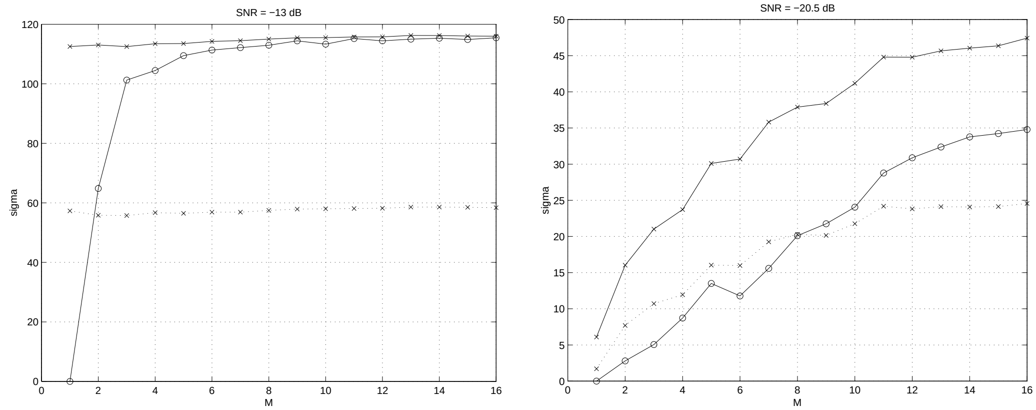
Note that it is not necessary to calibrate the array to achieve a detection using FXE. It is, however, necessary to calibrate the array to determine the direction from which the signal arrived. In practice, it is anticipated that at least a rough calibration could be maintained, so at least a coarse fix could be obtained using the primary eigenvector. Also (and unlike BF), FXE is equally sensitive to the entire FOV, with no losses due to cross-over between beams. Unlike MDL, FXE is not confounded by variations in receiver noise figures, although this comes at the expense of optimal sensitivity. Finally, FXE is immune to spectral baseline variations, because the detection metric is is normalized to the noise power in a given frequency bin, and is independent of the noise power in other frequency bins.

A shortcoming of FXE as described above is the use of the SVD to obtain the primary eigenvalue and eigenvector. An suitable alternative to the SVD is the "Power Method" [6], which can be implemented as follows:
1. Initialize $\mathbf{u}_1$ to be the sum of the snapshots used to form $\mathbf{R}^{(l)}$.
2. $\mathbf{q} \leftarrow \mathbf{R}^{(l)}\mathbf{u}_1$
3. $\mathbf{u}_1 \leftarrow \mathbf{q}/\|\mathbf{q}\|$
4. Repeat steps 2 and 3 $P$ times.
5. $\lambda_1^{(l)} = \mathbf{u}_1^H \mathbf{R}^{(l)} \mathbf{u}_1$

This is an iterative method which yields very high accuracy, even with $P$ on the order of $M$. The result is that the primary "eigenpair" can be computed using on the order of $N^2$ operations, as opposed to $N^3$ operations required for the SVD.

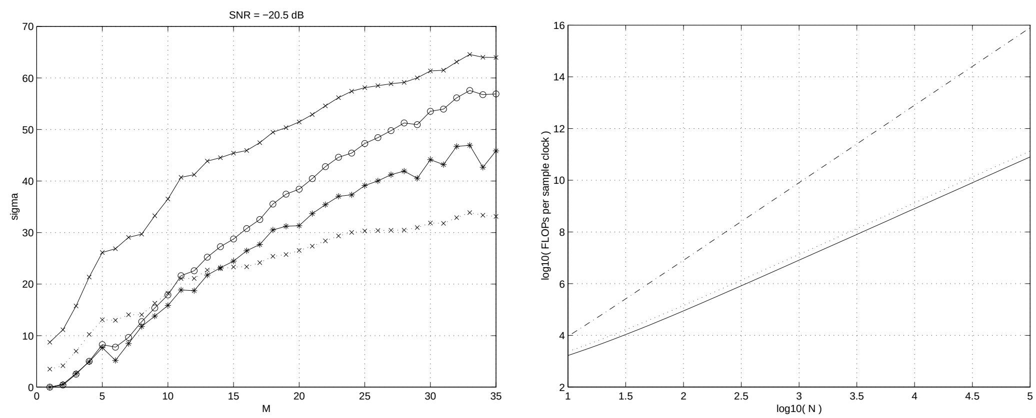
## IV. Performance Comparison

In this section, the performance of BF and FXE are compared using simulation. In this simulation, a single tone is incident on an array of $N = 16$ elements. The noise per element is independent and identically-distributed ("i.i.d.") white Gaussian noise. The FFT size is $L$ =16384. Figure 1(a) shows the performance when the signal-to-noise ratio (SNR) is $-13$ dB (+29 dB within the bin). The performance is quantified in terms of $\sigma$ obtained in the correct bin as a function of the number FFT blocks ($M$) processed. For the BF algorithm, the results from each successive PSD calculation are added to the sum of the previous results before computing the detection metrics. Thus, all methods are expected to improve with increasing $M$. In this example, we see that BF significantly outperforms FXE for low $M$; however, within about $M = 5$ iterations, the performance of BF and FXE is about the same.




(a) (b)

Fig. 1. (a) SNR = $-13$ dB. $\times$, *solid*: BF assuming full beam gain. $\times$, *dash*: BF assuming half-power beam gain. $\circ$: FXE. (b) Same as (a), with SNR equal to $-20.5$ dB.




(a) (b)

Fig. 2. (a) Same as Figure 1(b), now including the Power Method version of FXE ($*$) and showing results for larger $M$. (b) FLOPs per sample clock for each method as a function of number of antennas ($N$). *Solid:* BF, *Dash-Dot:* FXE (SVD), *Dot:* FXE (Power Method). $L = 16384$ and $M = P = 5$.

Also shown in Figure 1(a) is the detection performance for BF when the tone arrives from a cusp between beams. It is assumed that the cusp is at the half-power point, so the SNR delivered at the beam output is 3 dB less than the previous example in which we assumed that the tone's DOA was coincident with the beam maximum. Note that in this case, FXE outperforms BF for $M \geq 2$.

Figure 1(b) shows the same experiment repeated for SNR=$-20.5$ dB. Here, we see that FXE starts off slightly worse than BF at half-maximum gain, but at $M = 8$ crosses over and by $M = N$ is significantly better than BF at half-maximum gain, but still not as good as BF at maximum gain. This underscores an important point that FXE is in fact handicapped in it's detection performance by not having *a priori* pointing information. In other words, FXE's performance relative to BF suffers somewhat at low SNR because FXE is in some sense having to estimate a steering vector jointly with a detection metric, whereas BF benefits from having the array gain provided with no additional effort required. Figure 2(a) shows the same experiment, but now including FXE with the SVD replaced with the Power Method with $P = M$. Note that there is a slight degradation, but that either version of FXE outperforms BF at the half-power points for sufficiently large $M$.

## V. Computational Burden

The computational burden of the various algorithms can be estimated using the following rules for counting operations (FLOPs): (1) A complex scalar plus a complex scalar is 2 FLOPs, (2) A complex scalar times a complex scalar is 6 FLOPs, (3) A complex FFT of length $L$ is $6L\log_2 L$ FLOPs, and (4) An SVD of an $N \times N$ hermitian matrix is $40N^3$ FLOPs. From these rules, one finds that the inner product of 2 complex length-N vectors is $8N-2$ FLOPs, and that the product of an $N \times N$ matrix with a vector is $8N^2-2N$ FLOPs. Using these guidelines, one can estimate the number of FLOPs required to process $M$ blocks of $LN$ samples from the array for a single iteration of detection. These estimates include all operations up to and including computation of the integrated PSDs (in BF) and computation of the eigenvalues/eigenvectors (in FXE). The FLOP count for BF is:

$$8LMN^2 + 2(3M\log_2 L + M + 1)LN \tag{1}$$

and for FXE (using SVD) is:

$$40LN^3 + (4M-1)LN^2 + (6M\log_2 L + 4M + 1)LN \tag{2}$$

and for FXE (using the Power Method) is:

$$(8P + 4M + 7)LN^2 + (12P + 4M + 6M\log_2 L + 5)LN + 2(P+1)L\ . \tag{3}$$

The results for $L = 16384$ and $M = P = 5$ are shown in Figure 2(b). Whereas FXE with SVD requires orders of magnitude more FLOPs than BF, FXE using the Power Method requires only slightly more FLOPs than BF.

## VI. Pulse Detection Algorithm (TXE)

To detect undispersed pulses as opposed to tones, FXE can be modified as follows:

1. Divide the data record up into $L$ blocks of $M$ array output vectors (snapshots) each.
2. For each block $l$, compute the $(N^2+N)/2$ non-redundant cross-products between elements, summing over the $M$ results per block. This results in $L$ observations, each summarized by a covariance matrix $\mathbf{R}^{(l)}$.
3. Compute $\lambda_1^{(l)}$, the primary (largest) eigenvalue of $\mathbf{R}^{(l)}$ for each block, using SVD or the Power Method.
4. Compute $d_l = \lambda_1^{(l)}/(\mathrm{Tr}\{\mathbf{R}^{(l)}\} - \lambda_1^{(l)})$ for each block. $d_l$ expressed in standard deviations $\sigma$ from the mean across the remaining blocks.
5. A detection is declared if $d_l$ for any block is greater than some threshold.
6. The steering vector associated with a detection is the eigenvector of $\mathbf{R}^{(l)}$ associated with $\lambda_1^{(l)}$ for the block $l$ in which the detection occurred.

Note that the only difference from FXE is that the initial "channelization" of data is in terms of time segments, as opposed to frequency bins. For this reason we refer to this algorithm as "TXE". Just as FXE is robust to variations in spectral baselines, TXE is robust to slow (relative to the block length) changes in time-domain total power.

## Acknowledgments

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## References

[1] J. Tarter, J. Dreher, S.W. Ellingson, and W.J. Welch, "Recent Progress and Activities in the Search for Extraterrestrial Intelligence (SETI)," Chapter 36 of *Review of Radio Science, 1999-2002*, W. Ross Stone (Ed.), IEEE Press/John Wiley, 2002.

[2] A.O. Benz and G. Paesold, "A Search for Prompt Radio Emission of Gamma-Ray Bursts," *Astronomy and Astrophysics*, Vol. 329, 1998, pp. 61–67.

[3] A. Kinkhabwala and S.E. Thorsett, "Multifrequency Observations of Giant Radio Pulses from the Millisecond Pulsar B1937+21," *Astrophysical J.*, Vol. 535, No. 1, May 2000, pp. 365–72.

[4] D.H. Johnson and D.E. Dudgeon, *Array Signal Processing: Concepts and Techniques*, Prentice-Hall, 1993.

[5] M. Wax and T. Kailath, "Detection of Signals by Information Theoretic Criteria," *IEEE Trans. Acoustic, Speech, and Signal Processing*, Vol. ASSP-33, pp. 387–392, Apr. 1985.

[6] G.H. Golub and C.F. Van Loan, *Matrix Computations*, 3rd. Ed., The Johns Hopkins University Press, 1996.